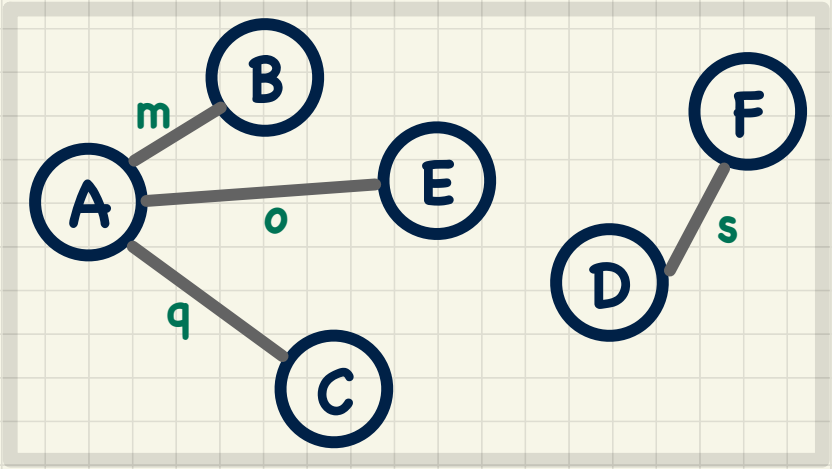
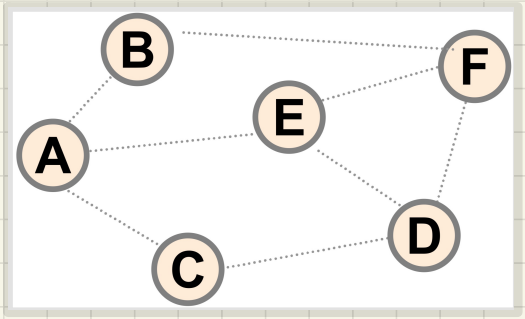
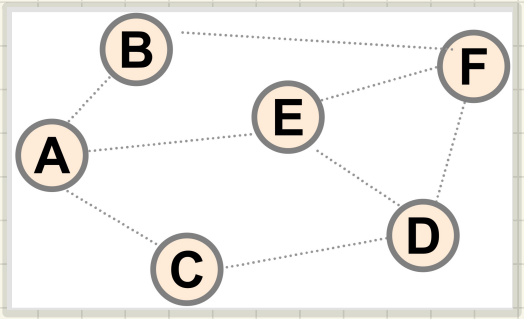
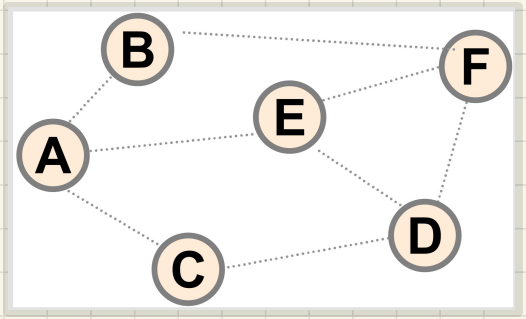
Graph: Forests and Trees

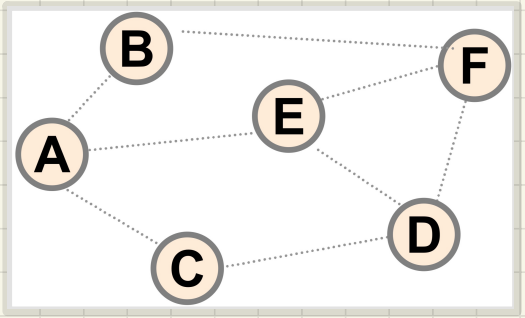


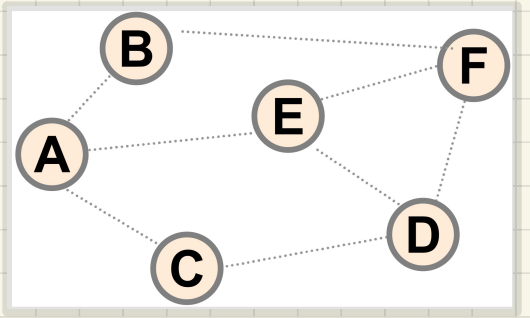
Graph: Spanning Trees





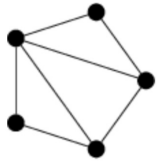




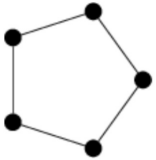


Graph: Exercises

Given a graph



Which one of the following is a ***spanning tree***?



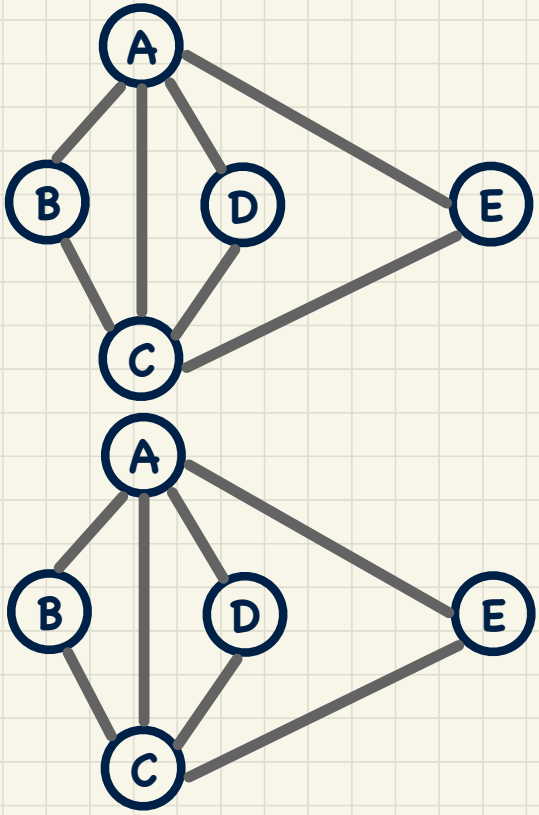
(a)

(b)

(c)

Graph Traversals: Definition & Applications

Efficient Traversal of Graph G:



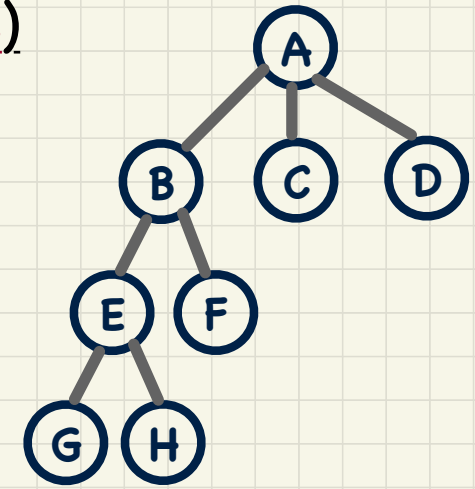
Applications:

- **Reachable** Vertices from $v \in V$
- A **path** between $\{u, v\} \subseteq V$
- The **minimum path** between $\{u, v\} \subseteq V$
- Is G **connected**?
- Compute a **spanning tree** of a connected G.
- Compute the **connected components** of G.
- If G is cyclic, return a **cycle**.

Graph Traversal: Depth-First Search (DFS)

A ***Depth-First Search*** (***DFS***) of graph G = (V, E),
starting from some vertex $v \in V$, proceeds along a **path** from v.

- The **path** is constructed by following **an** ***incident edge***.
- The **path** is extended **as far as possible**, until **all** ***incident edges*** lead to vertices that have already been ***visited***.
- Once the **path** originated from v **cannot be extended further**, ***backtrack*** to the **latest** vertex whose ***incident edges*** lead to some ***unvisited*** vertices.



Q. When a graph is a tree,
what kind of tree traversal does it correspond to?

Q. What data structure should be used to
keep track of the visited nodes?

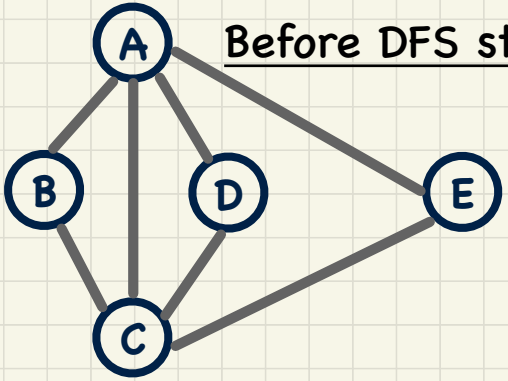
Depth-First Search (DFS): Marking Vertices & Edges

Before the ***DFS*** starts:

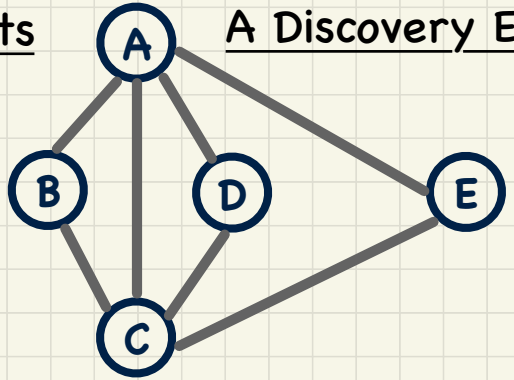
- All vertices are ***unvisited***.
- All edges are **unexplored/unmarked**.

Over the course of a ***DFS***, we **mark** vertices and edges:

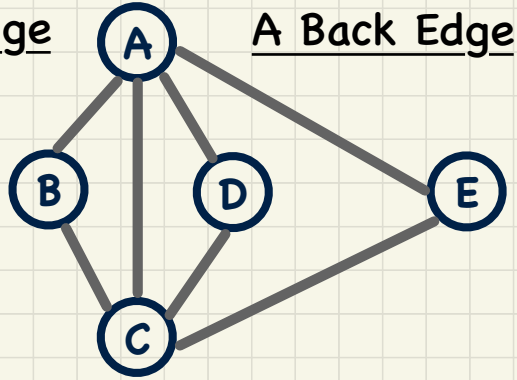
- A vertex *v* is marked ***visited*** when it is **first** encountered.
- Then, we iterate through each of *v*'s **incident edges**, say *e*:
  - If edge *e* is already **marked**, then skip it.
  - Otherwise, mark edge *e* as:
    - A ***discovery*** edge if it leads to an ***unvisited*** vertex
    - A ***back*** edge if it leads to a ***visited*** vertex (i.e., an ancestor vertex)



Before DFS starts



A Discovery Edge



A Back Edge